Proximal MCMC– Towards a Sparse Earth Model

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1. Introduction

We use proximal Markov Chain Monte Carlo (PxMCMC) to build global Rayleigh wave phase velocity maps, promoting sparsity in a spherical wavelet basis. This method allows us to fully quantify the uncertainties in our maps.

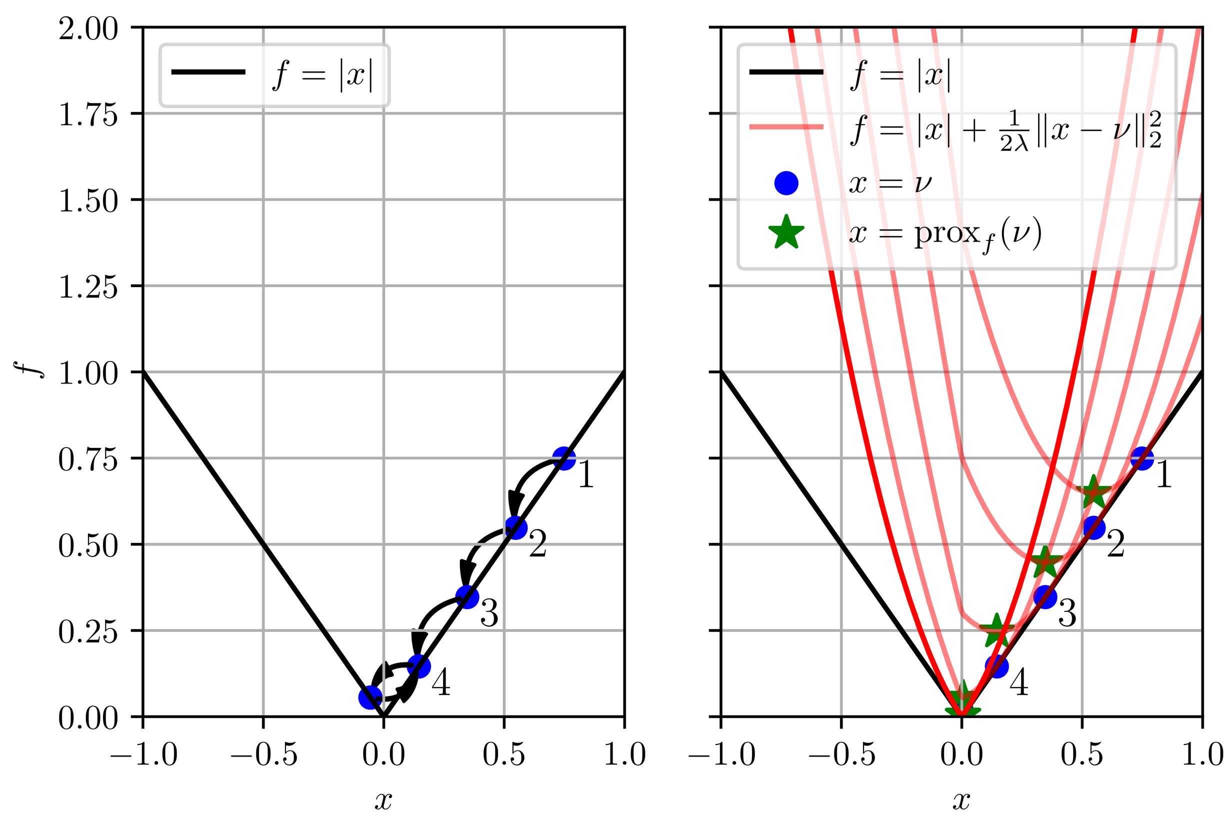
PxMCMC is a sampling algorithm designed to efficiently sample a high-dimensional parameter space while allowing for the non-differentiable priors needed to promote sparsity.

Other high-dimensional sampling methods (e.g. Hamiltonian Monte Carlo) are based on gradients, restricting the type of prior information that can be used.

Using Fourier-Laguerre wavelets (*flaglets*) along the radial line, we aim to use PxMCMC to create an entirely probabilistic upper mantle shear wave speed model.

2. Proximal Mappings

Proximal mappings behave similarly to gradients, in that they move points towards the max/min of the function. They are, however, more general than gradients, and can be applied to non-differentiable functions



3. Proximal Markov Chain Monte Carlo

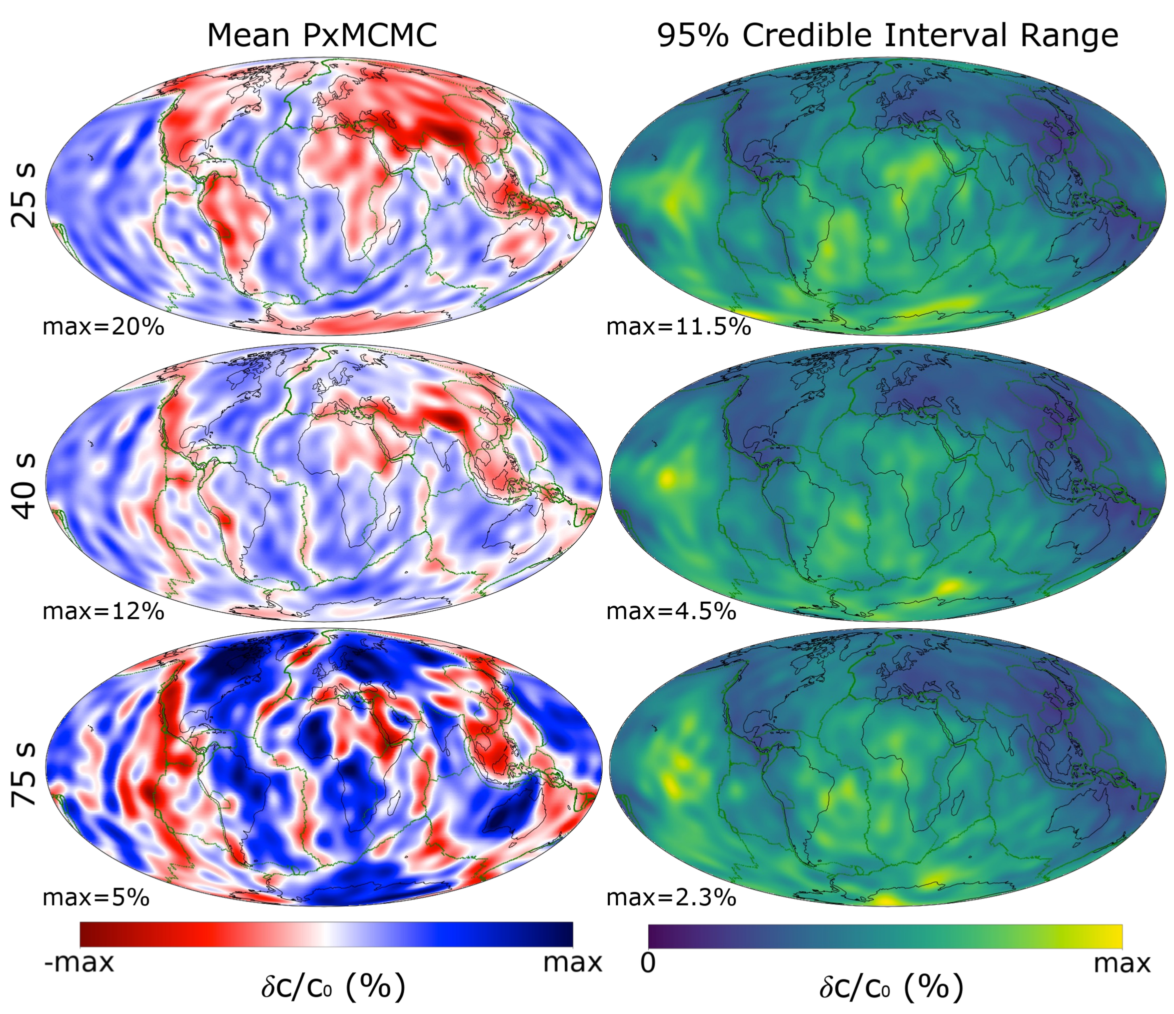
Modified from the *Unadjusted Langevin Equation*, which moves around the parameter space according to Langevin dynamics.

$$\boldsymbol{m}^{(i+1)} = \left(1 - \frac{\delta}{\lambda}\right)\boldsymbol{m}^{(i)} - \delta\nabla g + \frac{\delta}{\lambda}\mathrm{prox}_f^{\lambda}\left(\boldsymbol{m}^{(i)}\right) + \sqrt{\delta}\boldsymbol{w}^{(i)}$$

next chain sample gradient of likelihood prox of prior
current chain sample randomness

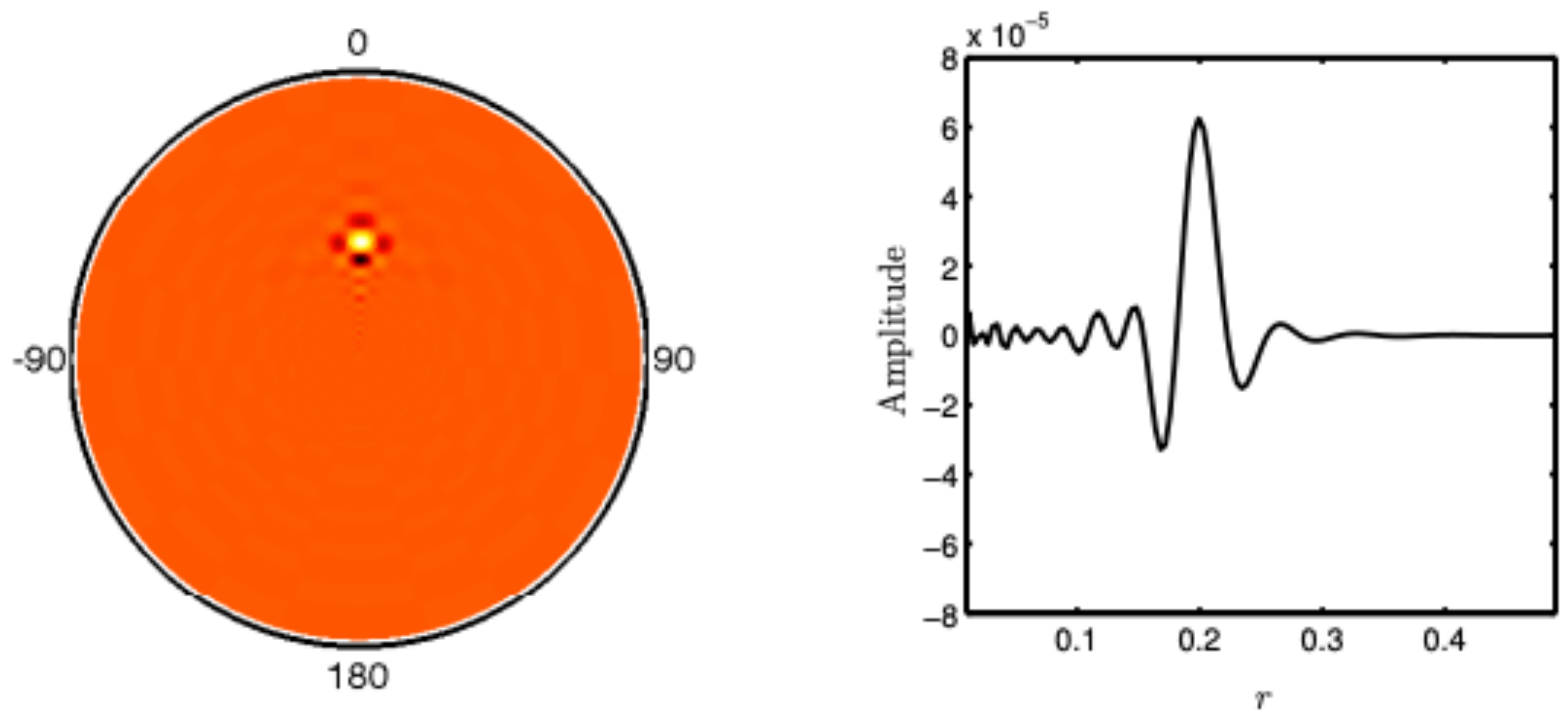
4. Rayleigh Wave Phase Velocity Maps

Our uncertainty maps (right) reflect the ray coverage, as well as artefacts. The fast streak off the coast of the western US is a known artefact resulting from not modelling anisotropy, and is highlighted in our uncertainties.



5. 3D Inversions with flaglets

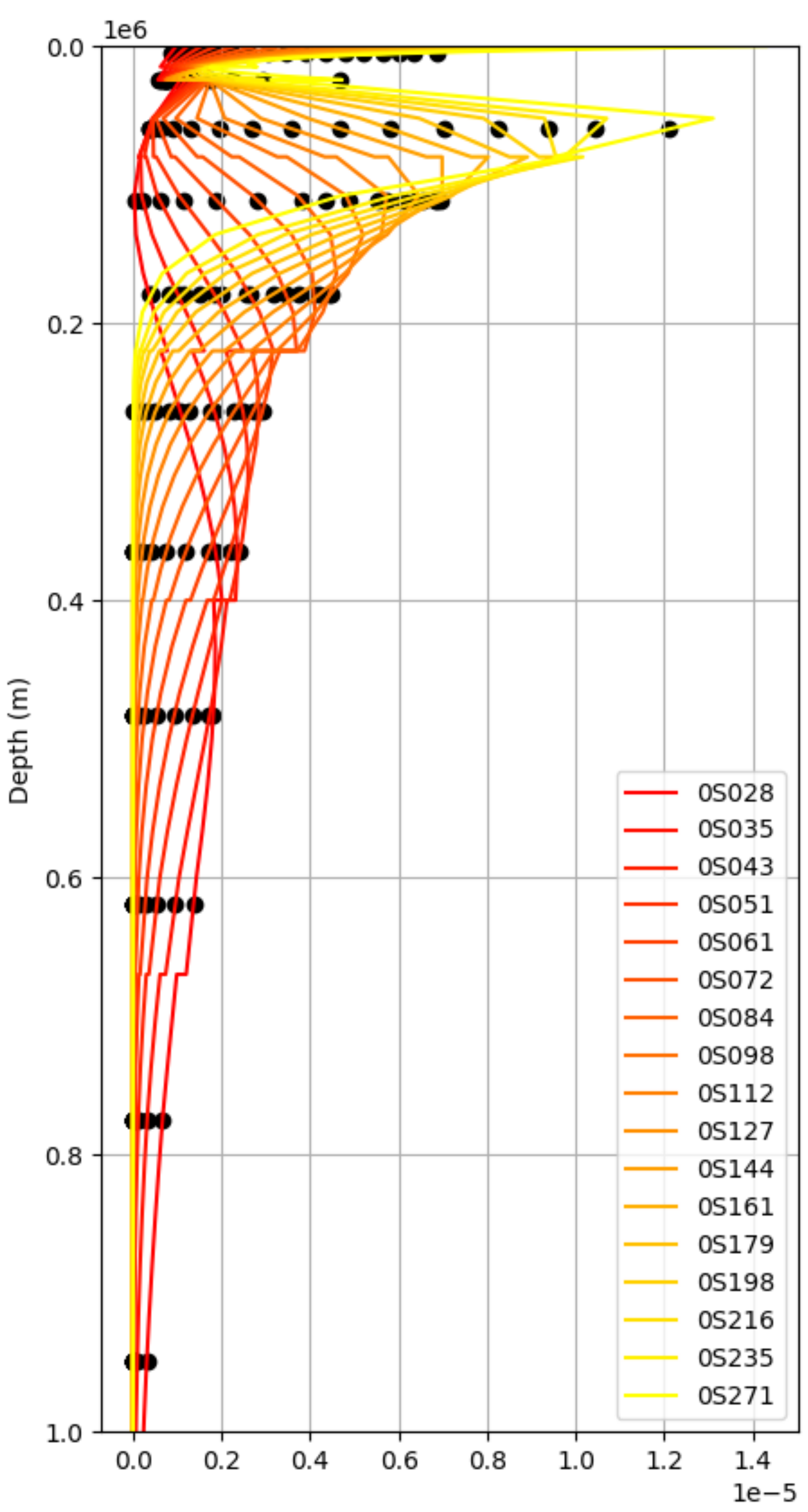
Fourier-Laguerre functions form a basis on the half-line [0,∞). Combining these with the spherical wavelets gives wavelets on the 3D ball. The next step is to invert the phase velocity maps using 3D wavelets to get *Vs* in the upper mantle.



Discretising the radial line at the roots of the Fourier-Laguerre creates a sampling theorem with denser sampling at one end.

The sensitivity of Rayleigh wave phase velocity to *Vs* is more variable nearer to the surface, where we can make use of the denser sampling.

We have phase velocity maps for 17 wave periods, giving sensitivity down to about 600 km depth



5. Going Forward

Spherical Fourier Laguerre transformations are computationally expensive. Current work to perform these on GPUs.

Need to consider how to parameterise mantle discontinuities at 410 km and 660 km.

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M. Pereyra (2016) *Proximal Markov chain Monte Carlo algorithms,* Statistics and Computing
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