Proximal Markov Chain Monte Carlo – Towards a Sparse Earth Model

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1. Introduction

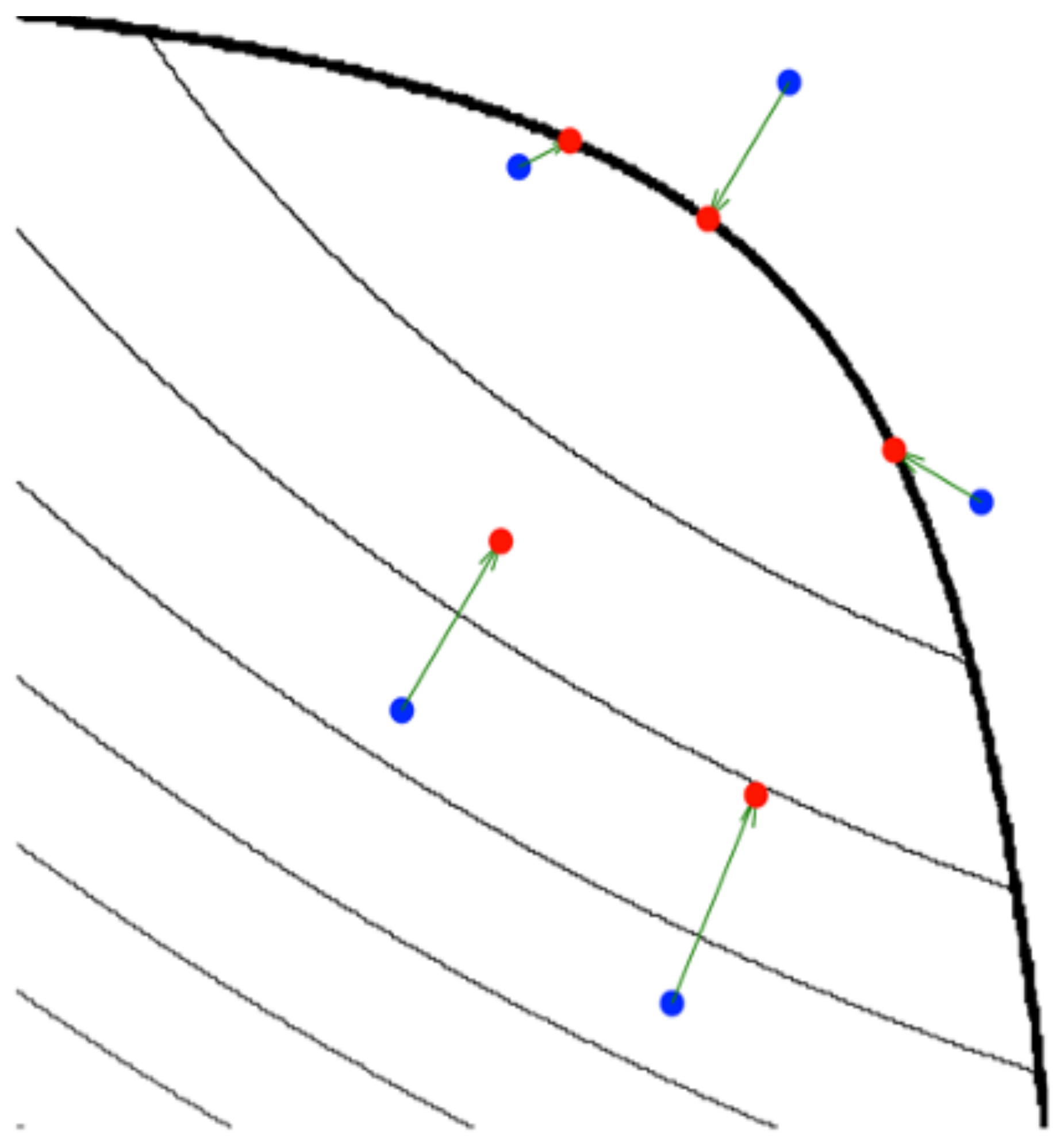
We use proximal Markov Chain Monte Carlo (PxMCMC) to build global Rayleigh wave phase velocity maps, promoting sparsity in a spherical wavelet basis. This method allows us to fully quantify the uncertainties in our maps.

PxMCMC is a sampling algorithm designed to efficiently sample a high-dimensional parameter space while allowing for the non-differentiable priors needed to promote sparsity.

Other high-dimensional sampling methods (e.g. Hamiltonian Monte Carlo) are based on gradients, restricting the type of prior information that can be used.

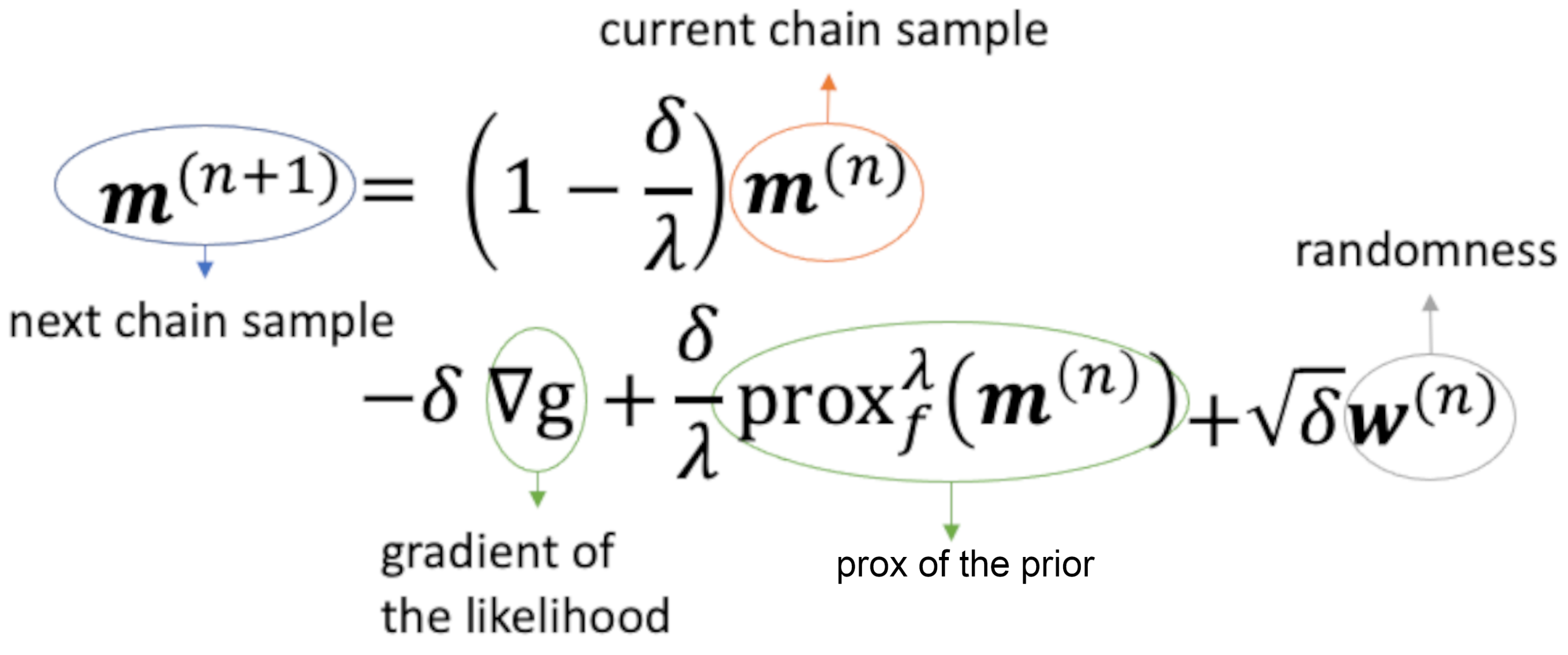
2. Proximal Mappings

Proximal mappings behave similarly to gradients, in that they move points towards the max/min of the function. They are, however, more general than gradients, and can be applied to non-differentiable functions



Parikh & Boyd, 2013

3. Proximal Markov Chain Monte Carlo



$$\boldsymbol{m}^{(n+1)} = \left(1-\frac{\delta}{\lambda}\right)\boldsymbol{m}^{(n)} - \delta\,\nabla g + \frac{\delta}{\lambda}\mathrm{prox}_f^{\lambda}\left(\boldsymbol{m}^{(n)}\right) + \sqrt{\delta}\boldsymbol{w}^{(n)}$$

$$g(\boldsymbol{m}) = \frac{1}{2\sigma^2}\|\boldsymbol{d}-\boldsymbol{A}\boldsymbol{m}\|_2^2 \qquad f(\boldsymbol{m}) = \mu\|\boldsymbol{m}\|_1$$

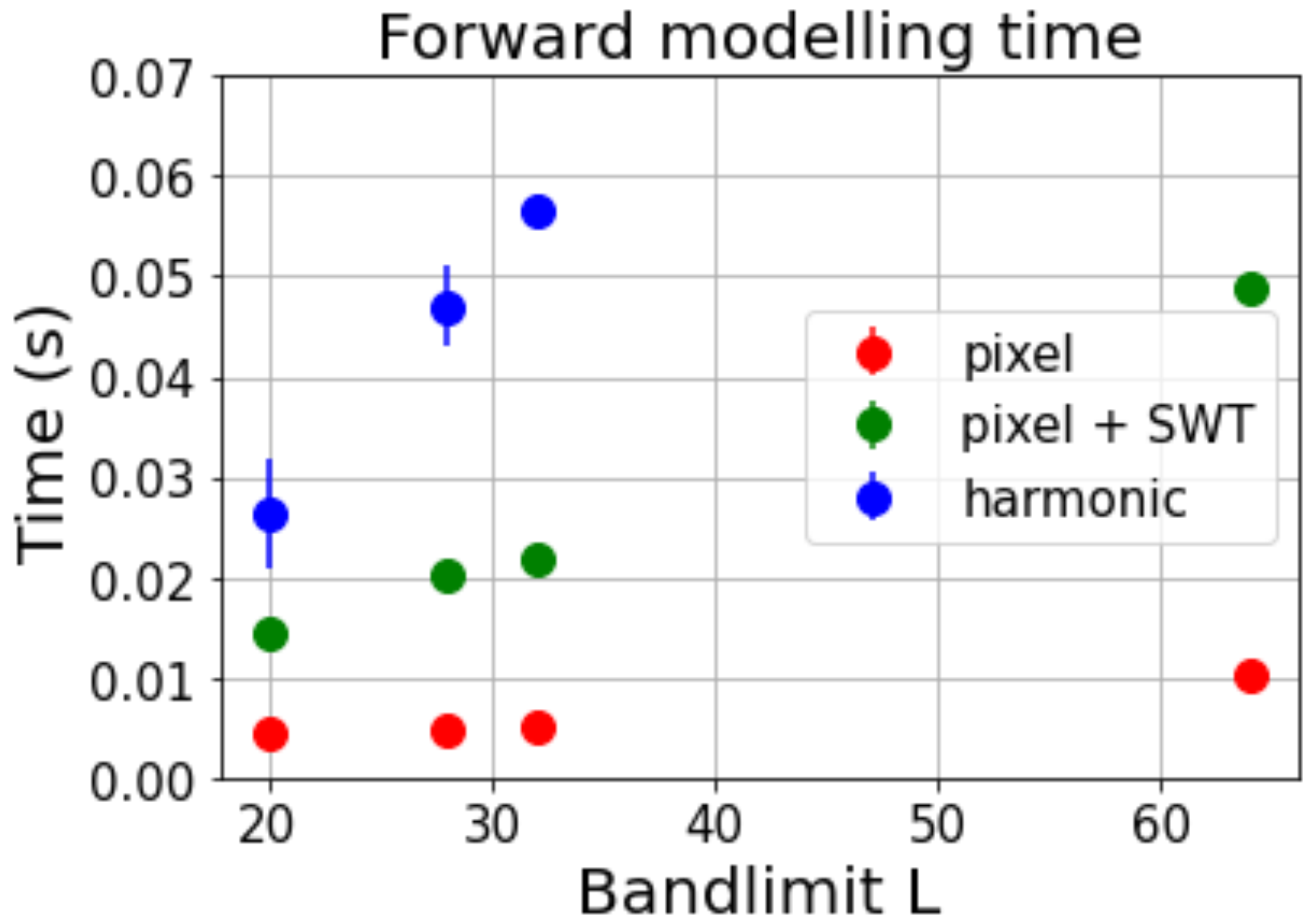
$$\Longrightarrow \nabla g = \boldsymbol{A}^{\dagger}(\boldsymbol{A}\boldsymbol{m} - \boldsymbol{d})/\sigma^2 \qquad \Longrightarrow \mathrm{prox}_f^{\lambda}(\boldsymbol{m}) = \mathrm{soft}_{\mu\lambda}(\boldsymbol{m})$$

$\mathrm{soft}_{\mu\lambda}$ is the soft thresholding operator, which is easy and fast to calculate.

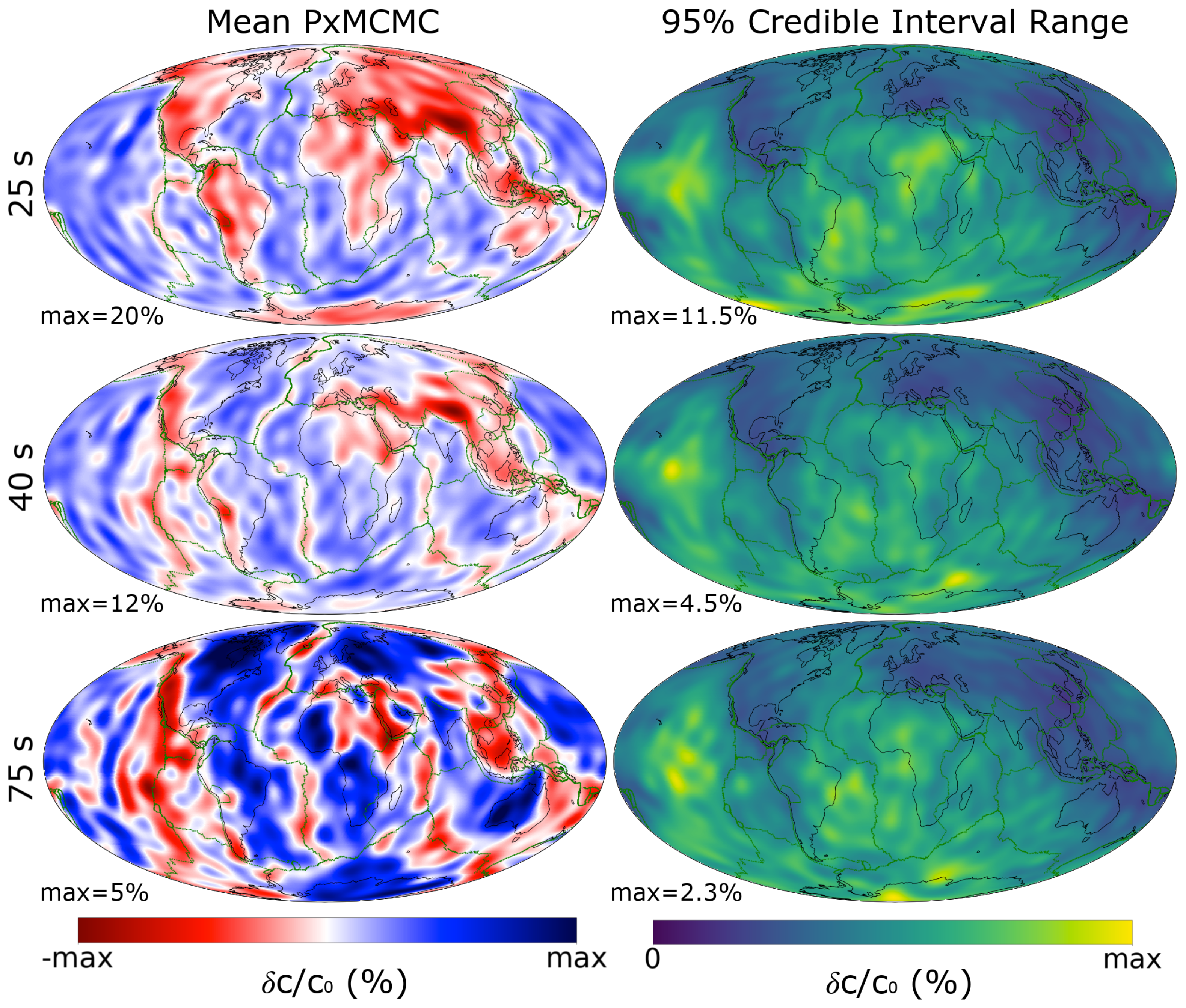
Pereyra, 2016; Combettes & Pesquet, 2011

4. Fast great circle path integration

Calculate the path integral as a weighted sum in pixel space, rather than using the more common harmonic formulation. The weighted sum becomes an extremely sparse matrix multiplication, speeding up the forward modelling up to 15 times.



5. Results



Our uncertainty maps (right) reflect the ray coverage, as well as artefacts. The fast streak off the coast of the western US is a known artefact resulting from not modelling anisotropy, and is highlighted in our uncertainties.

6. Conclusions

Our method produces good results and physically reasonable uncertainties of 2D seismic images.

Special consideration needs to be given to the forward modelling step to ensure computational feasibility.

Extending this to 3D will produce a sparse Earth model.

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A. Marignier, J. D. McEwen, A. M. G. Ferreira, T. Kitching (2021) *Posterior sampling for inverse imaging problems on the sphere*, IEEE TIP (under review)
N. Parikh, S. Boyd (2014) *Proximal algorithms*, Foundations and Trends in Optimisation
M. Pereyra (2016) *Proximal Markov chain Monte Carlo algorithms*, Statistics and Computing
P. L. Combettes, J.-C. Pesquet (2011) *Proximal Splitting Methods in Signal* , Springer