Cosmological mass-mapping with trans-dimensional trees

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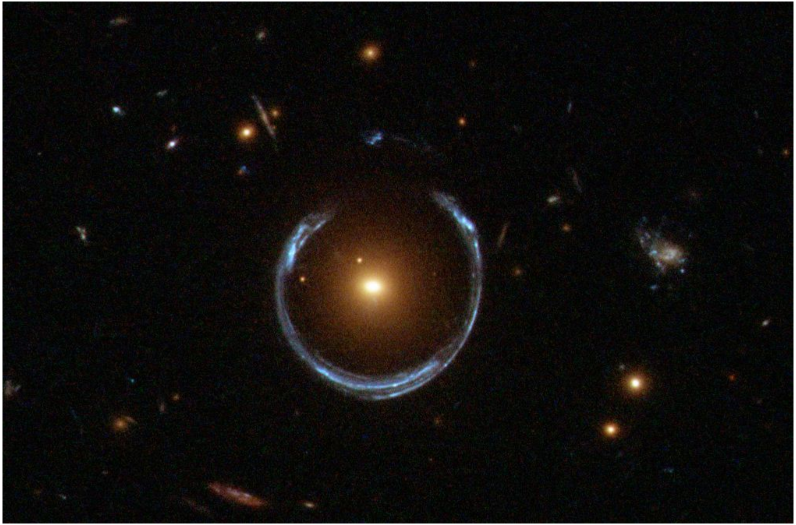
Gravitational Lensing



Our Aim

Create mass maps with full uncertainty quantification, via a Bayesian inversion

Promote sparsity in a wavelet basis

Bayesian Inversion

POSTERIOR – what we want

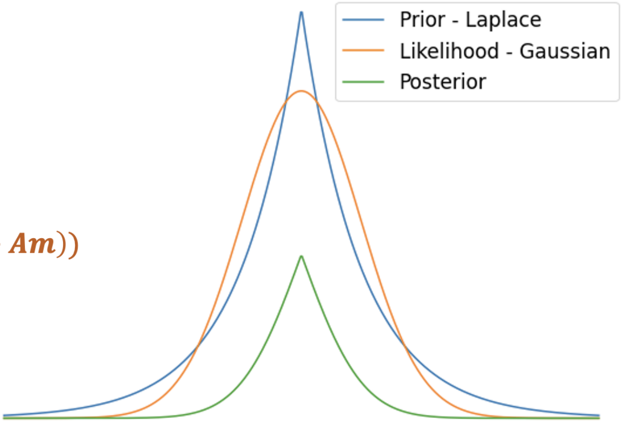
$p(\boldsymbol{m}|\boldsymbol{d}) \propto p(\boldsymbol{d}|\boldsymbol{m})p(\boldsymbol{m})$

LIKELIHOOD – what we have

$p(\boldsymbol{d}|\boldsymbol{m}) \propto \exp(-(\boldsymbol{d} - \boldsymbol{A}\boldsymbol{m})^T \boldsymbol{C}^{-1}(\boldsymbol{d} - \boldsymbol{A}\boldsymbol{m}))$

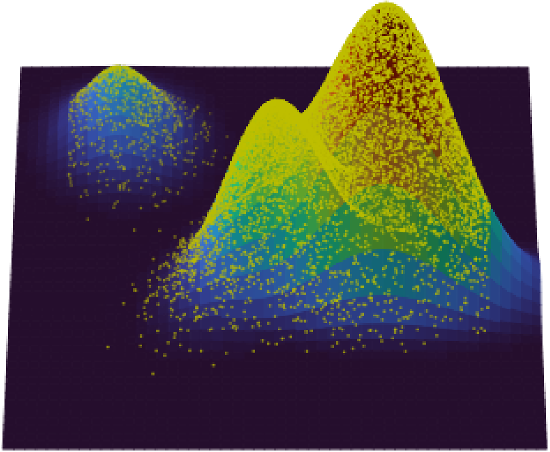
PRIOR – what we think

$p(\boldsymbol{m}) \propto \exp(-\mu|\boldsymbol{m}|)$



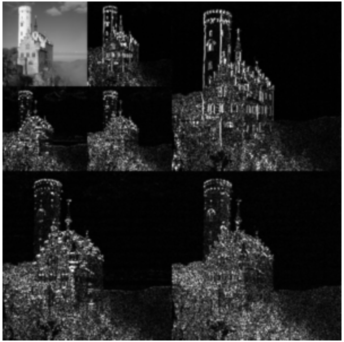
Posterior sampling

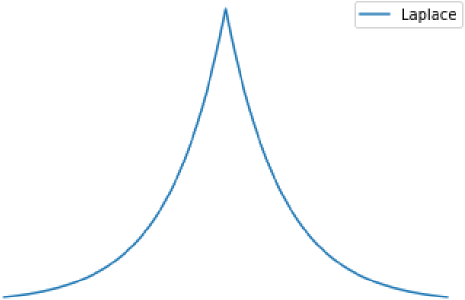
Repeatedly try points in parameter space and compare predictions with observed data



Wavelets and Sparsity

Natural images tend to be sparse in a wavelet basis, so we can use this as prior information





$256 \times 256 \Rightarrow 65,536$ wavelet coefficients $\Rightarrow$ Too many to sample!

Trans-dimensional Bayesian Inversion

Bayes Theorem

$$p(\theta|\mathbf{d}) \propto p(\mathbf{d}|\theta)p(\theta)$$

where θ is a k-dimensional vector of unknown model parameters (wavelet coefficients)
where ***k* is also unknown**

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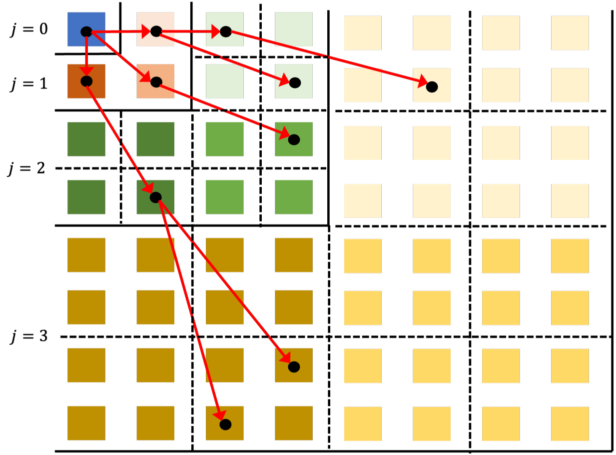
Generalising the common MCMC Metropolis-Hastings acceptance criteria gives

$$\alpha(\theta'|\theta) = \min\left\{1, \frac{p(\theta')p(\theta'|\mathbf{d})q(\theta|\theta')}{p(\theta)p(\theta|\mathbf{d})q(\theta'|\theta)}|\mathcal{J}|\right\}$$

where $\mathcal{J}$ is the Jacobian matrix of the transformations between parameter spaces

Very commonly used in seismic imaging

Wavelet Tree Parameterisation

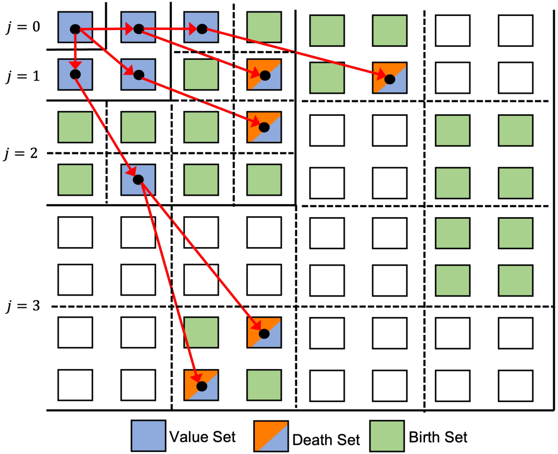


Trans-dimensional Trees

The parameter space can be divided up into three sets

1. The set of k active wavelet coefficients/tree nodes, who's value can change
2. Nodes that could possibly die
3. Nodes that could possibly be born

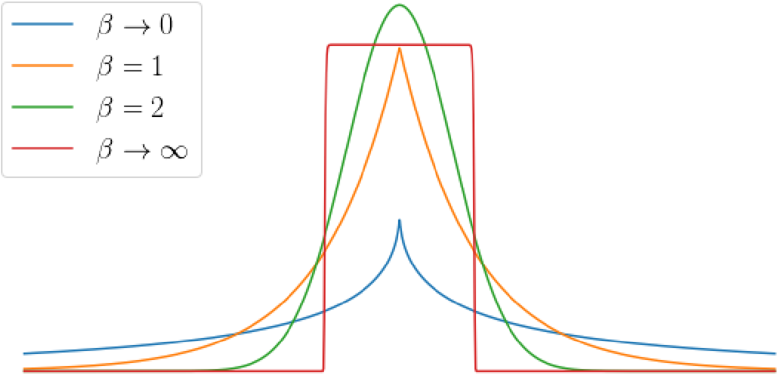
These each have their own proposal distribution $q(\theta|\theta')$



Prior on wavelet coefficients

The Generalised Gaussian Distribution (GGD)

$$f(x|\mu, \sigma, \beta) = \frac{\beta}{2\sigma\Gamma(\beta^{-1})} e^{\left(-\left|\frac{x-\mu}{\sigma}\right|^{\beta}\right)}$$



Mass-Mapping

Measurements of galaxy shapes $\rightarrow$ shear field $\gamma(\vec{\theta})$

Want convergence field $\kappa(\vec{\theta})$ (related to density)

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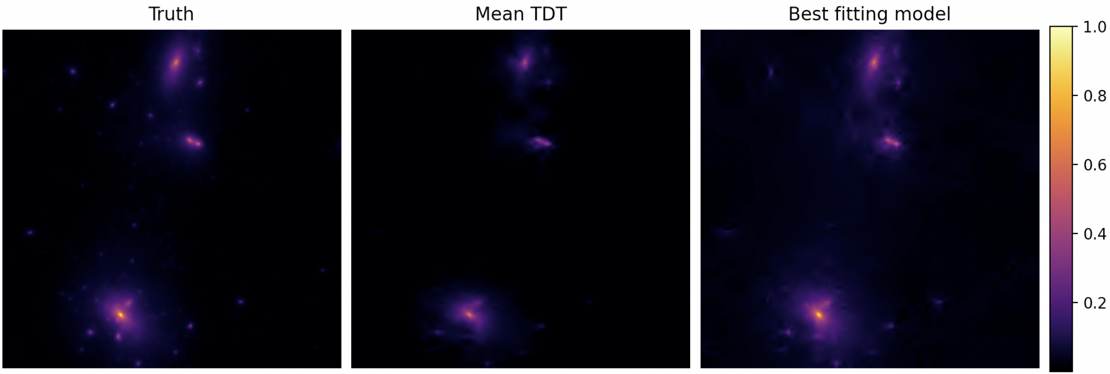
So our forward model is given by

$$\gamma = \mathbf{F}^{-1}\mathbf{D}\mathbf{F}\kappa$$

where $\mathbf{F}$ $(\mathbf{F}^{-1})$ is the (inverse) FFT, and $\mathbf{D}$ is the lensing kernel

Simple synthetic test



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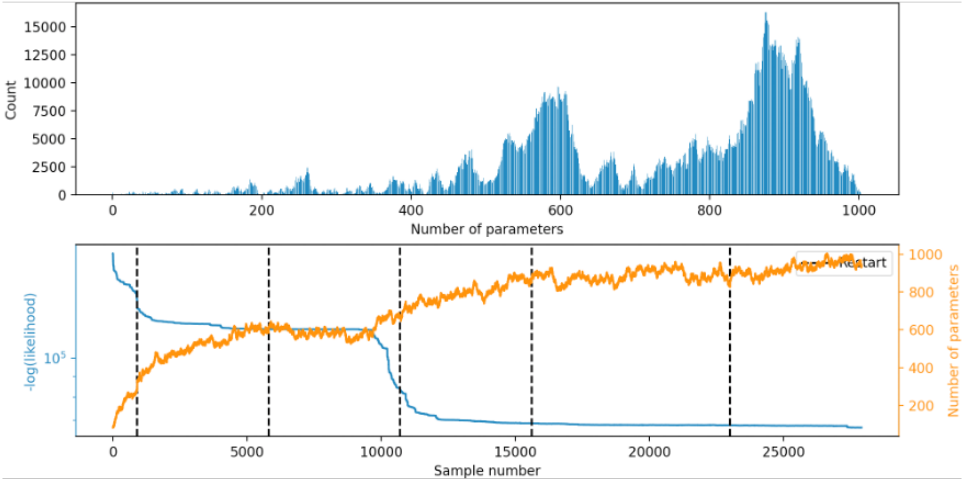

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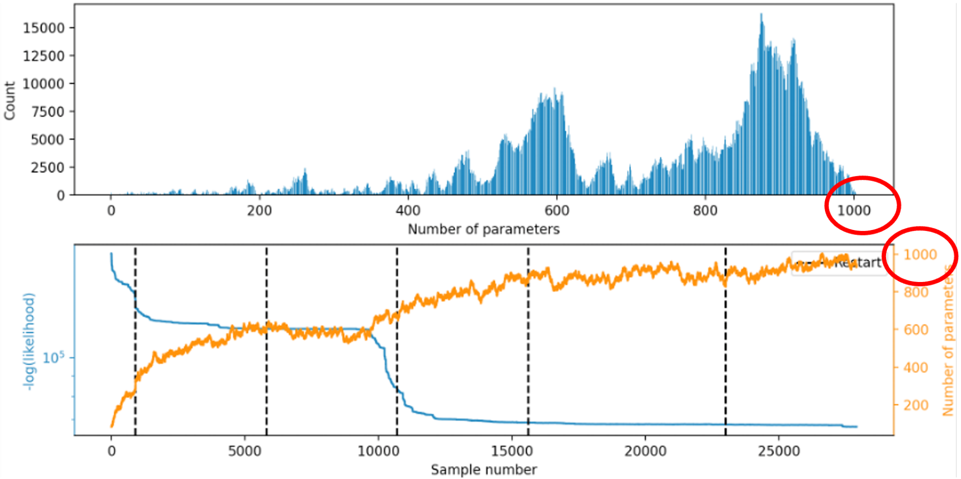
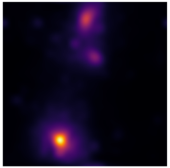
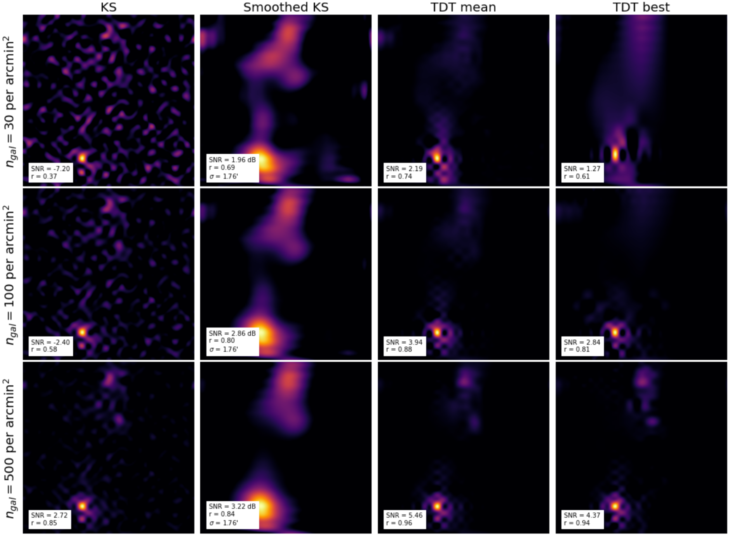


Compare with Kaiser-Squires





Conclusions

Trans-dimensional approach gives promising results on simulations

By slowly growing the parameter space, it is more efficiently sampled, making this high-dimensional problem computationally tractable

Currently working on real data inversions and uncertainty quantification

Hypothesis testing can give clues to the nature of dark matter